

# Viscous hydrodynamics — shear, bulk and numerical viscosity

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# Viscous hydro: a short summary for shear viscosity

--shear viscosity only

-2+1-d viscous hydro code individually developed by different groups:

Romatsche & Romatschke (INT), Song & Heinz (OSU), Dusling & Teaney (Stony)

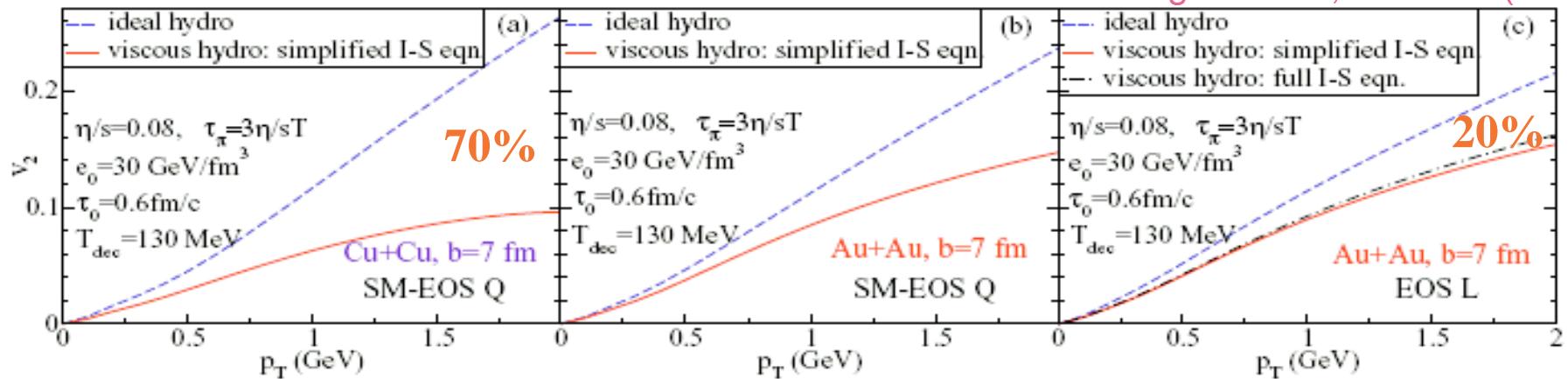
Huovinen & Molnar (Purdue), Chaudhuri (Kolkata, India)

- $v_2$  at RHIC is sensitive to even the minimum shear viscosity entropy ratio

- $v_2$  suppression from different groups ranges from 20% to 70%

-the above discrepancy was largely resolved by investigating effects from system size, EoS and different forms of I-S eqns. used

Song & Heinz, PRC 78 (2008)

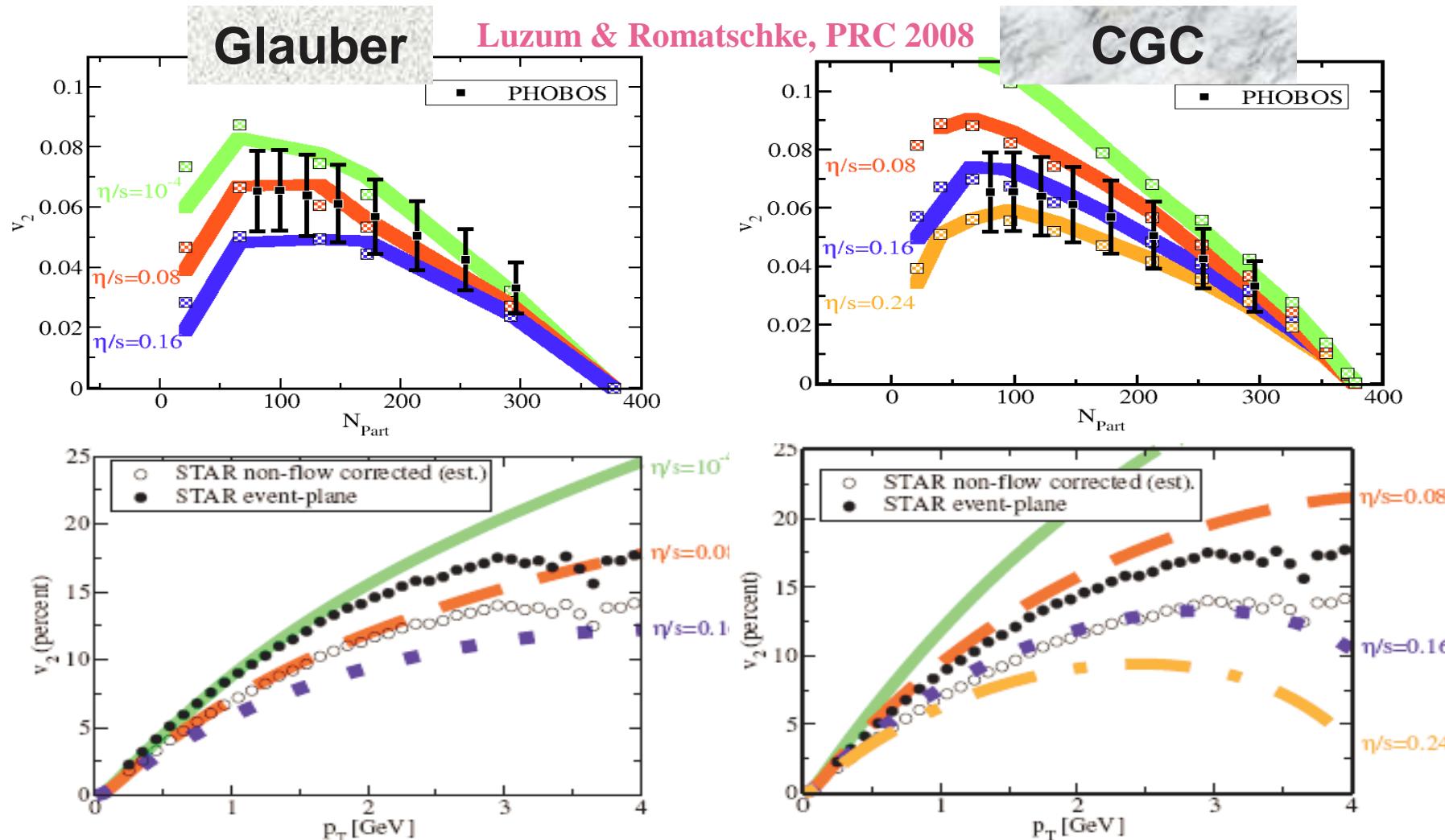


-Code checking within the TECHQM collaboration:

TECHQM webpage , Dusling talk in this meeting

-The first attempt to extract QGP shear viscosity from RHIC data: 2

Luzum & Romatschke, PRC 78 (2008)



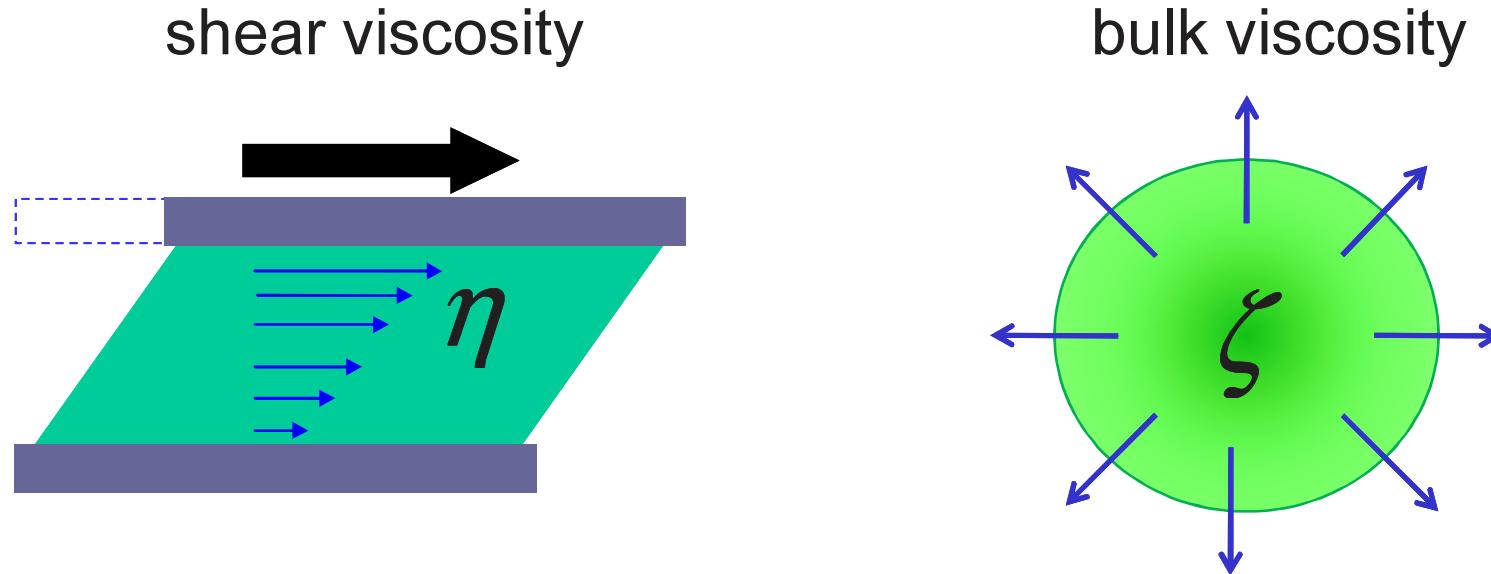
-Glauber vs.CGC ~100% effect on the extracted value of  $\eta/s$  , safely:  $\eta/s \leq 5 \times (1/4\pi)$

-a detailed extraction of shear viscosity entropy ratio requires:

-viscous late hadronic stage  
-non-equilibrium chemistry in HG } have been studied in ideal hydro

**bulk viscosity ?**

# Viscous hydro with shear & bulk viscosity



$$\partial_\mu T^{\mu\nu}(x) = 0 \quad T^{\mu\nu} = (e + p + \Pi) u^\mu u^\nu - (p + \Pi) g^{\mu\nu} + \pi^{\mu\nu}$$

$$\begin{aligned} \Delta^{\mu\alpha} \Delta^{\nu\beta} D \pi_{\alpha\beta} &= -\frac{1}{\tau_\pi} [\pi^{\mu\nu} - 2\eta \sigma^{\mu\nu}] - \frac{1}{2} \pi^{\mu\nu} \frac{\eta T}{\tau_\pi} \partial_\lambda \left( \frac{\tau_\pi}{\eta T} u^\lambda \right) \\ D\Pi &= -\frac{1}{\tau_\Pi} [\Pi + \zeta (\partial \cdot u)] - \frac{1}{2} \Pi \frac{\zeta T}{\tau_\Pi} \partial_\lambda \left( \frac{\tau_\Pi}{\zeta T} u^\lambda \right) \end{aligned}$$

(2<sup>nd</sup> order shear-bulk -mixing term (Muronga, Rischke) not included.) <sup>4</sup>

# Viscous hydro in 2+1-dimension

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad T^{\mu\nu} = (e + p + \Pi) u^\mu u^\nu - (p + \Pi) g^{\mu\nu} + \pi^{\mu\nu}$$

$$\begin{aligned}\Delta^{\mu\alpha} \Delta^{\nu\beta} D \pi_{\alpha\beta} &= -\frac{1}{\tau_\pi} [\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}] - \frac{1}{2} \pi^{\mu\nu} \frac{\eta T}{\tau_\pi} \partial_\lambda \left( \frac{\tau_\pi}{\eta T} u^\lambda \right) \\ D\Pi &= -\frac{1}{\tau_\Pi} [\Pi + \zeta(\partial \cdot u)] - \frac{1}{2} \Pi \frac{\zeta T}{\tau_\Pi} \partial_\lambda \left( \frac{\tau_\Pi}{\zeta T} u^\lambda \right)\end{aligned}$$

Bjorken approximation:  $(\tau, x, y, \eta)$  coordinates  $3+1 \Rightarrow 2+1$

--the transport equations for energy momentum tensor are explicit written as:

$$\begin{aligned}\frac{1}{\tau} \partial_\tau (\tau T^{\tau\tau}) + \partial_x (T^{\tau x}) + \partial_y (T^{\tau y}) &= -\frac{p + \Pi + \tau^2 \pi^{\eta\eta}}{\tau} \\ \frac{1}{\tau} \partial_\tau (\tau T^{xx}) + \partial_x ((T^{xx} - \pi^{xx}) v_x) + \partial_y ((T^{xy} - \pi^{xy}) v_y) &= -\partial_x (p + \Pi + \pi^{xx}) - \partial_y \pi^{xy} \\ \frac{1}{\tau} \partial_\tau (\tau T^{yy}) + \partial_x ((T^{yy} - \pi^{yy}) v_x) + \partial_y ((T^{xy} - \pi^{xy}) v_y) &= -\partial_y (p + \Pi + \pi^{yy}) - \partial_x \pi^{xy}\end{aligned}$$

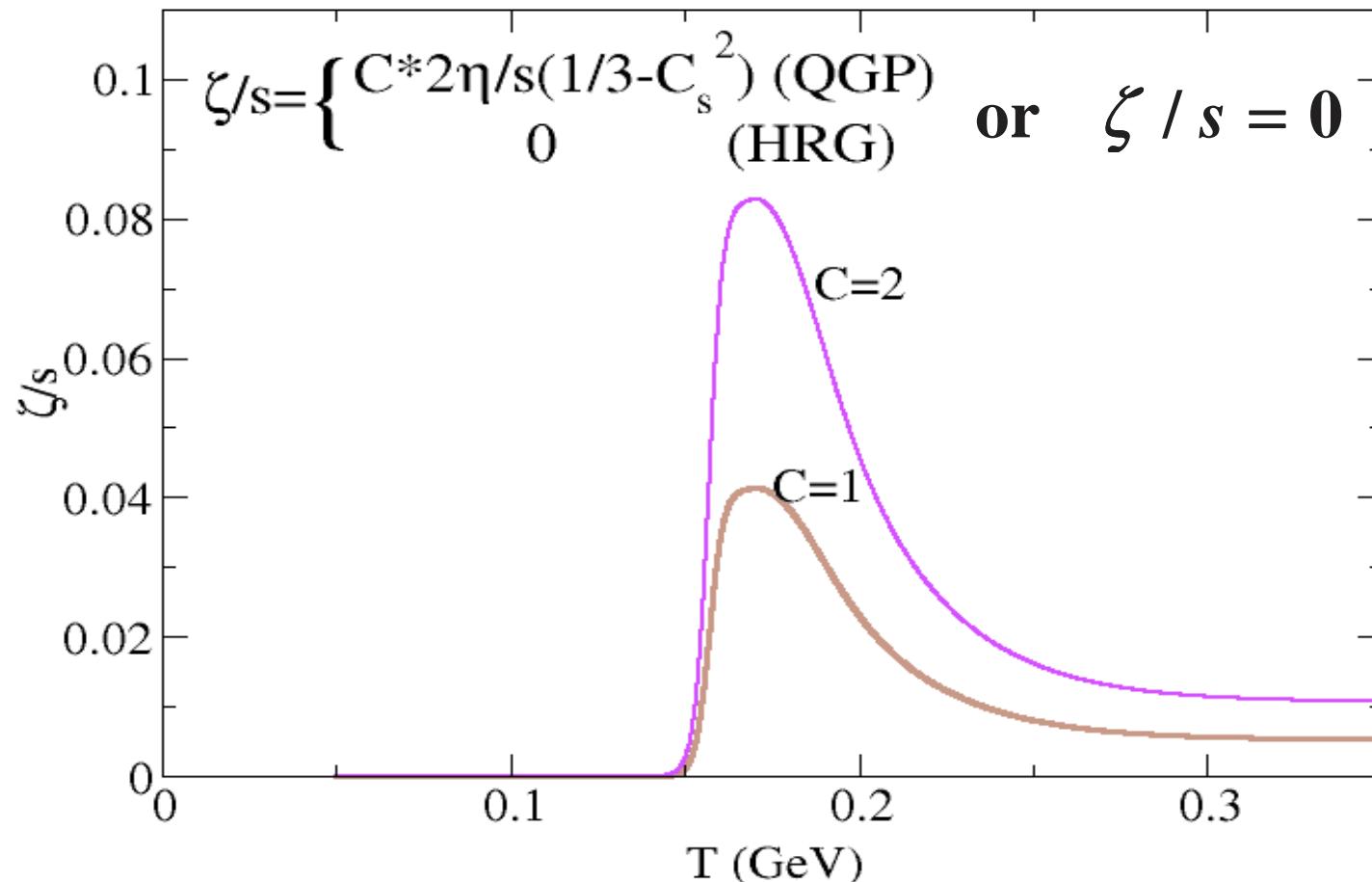
- shear tensor decelerate longitudinal expansion, but accelerate transverse expansion
- bulk pressure decelerates both longitudinal & transverse expansion (bulk pressure effectively softens the EoS near the QCD phase transition)

# Numerical Results

shear viscosity:

$$\eta / s = 0.08 \approx 1 / 4\pi, \quad \text{or} \quad \eta / s = 0$$

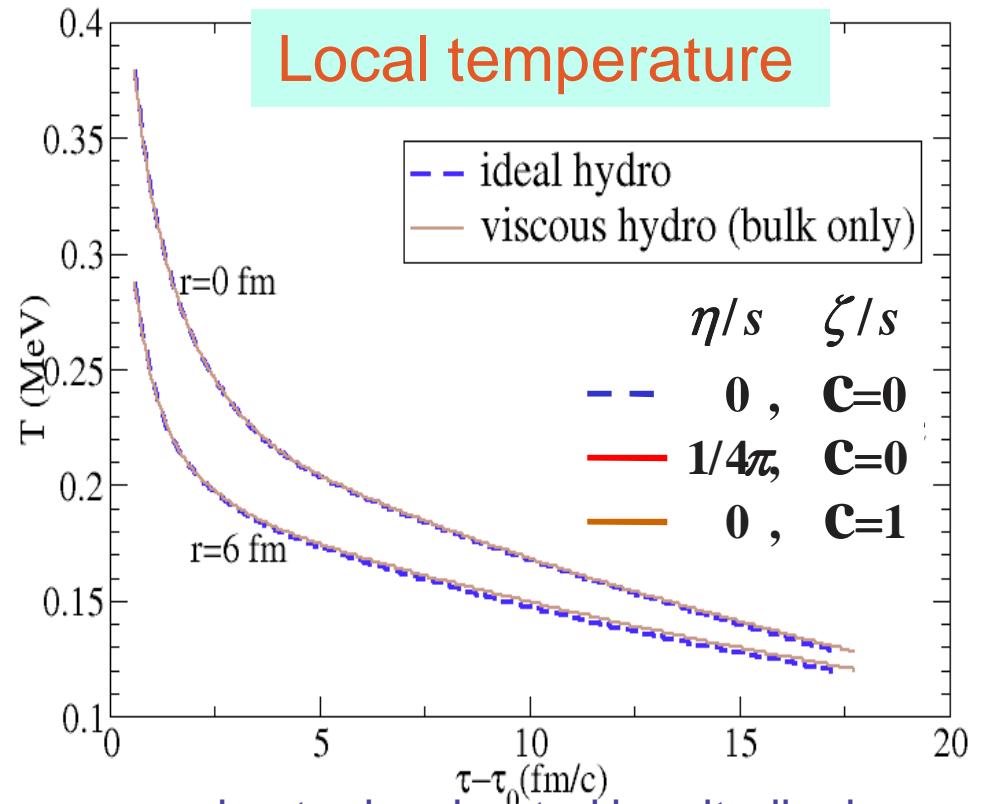
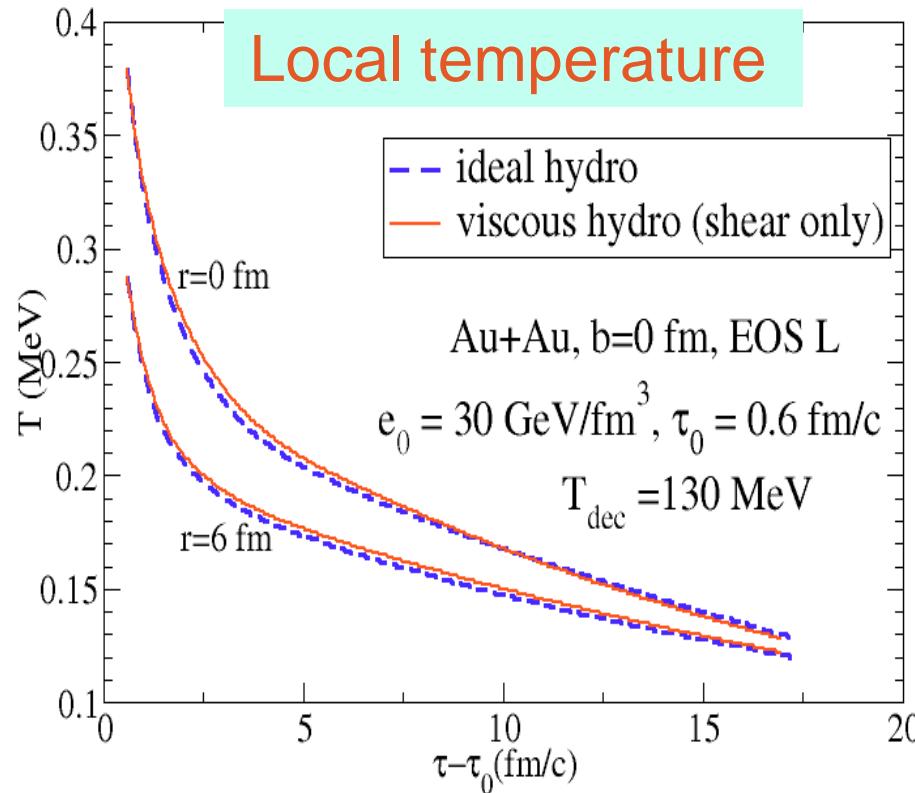
bulk viscosity:



# Shear viscosity vs. bulk viscosity (I)

The same Initial & final conditions:

— ideal hydro    viscous hydro-shear only    viscous hydro-bulk only



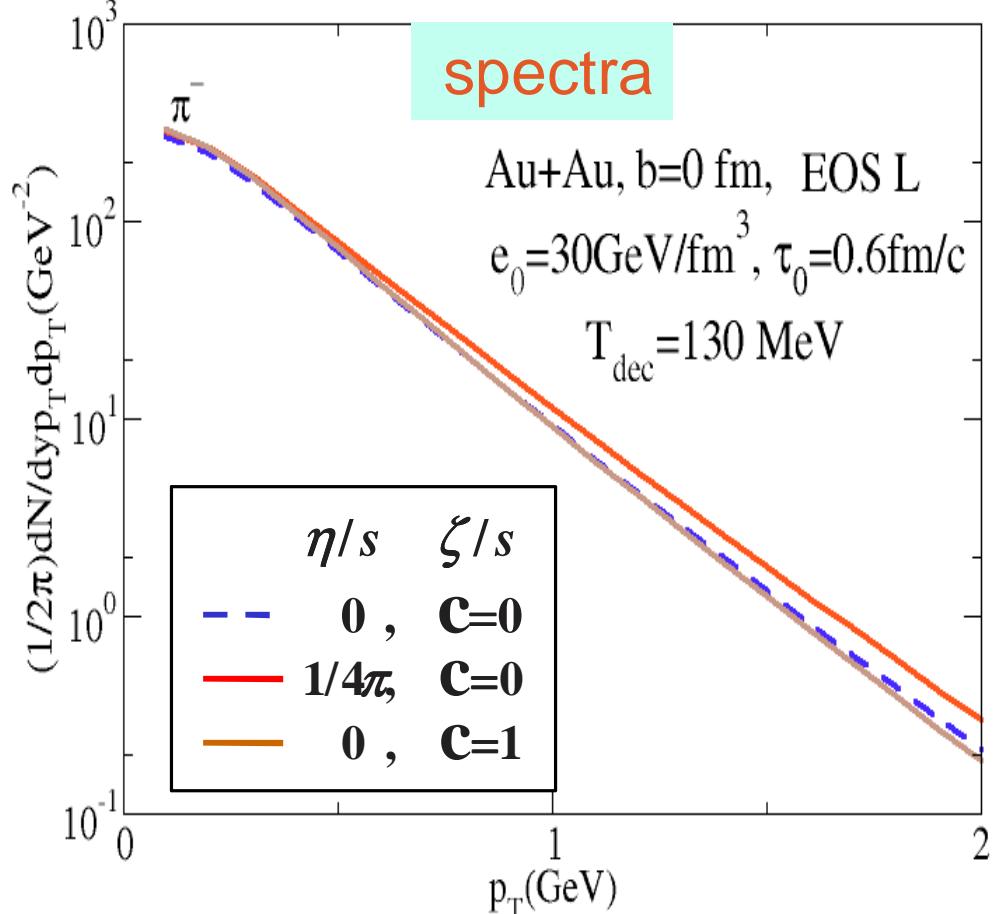
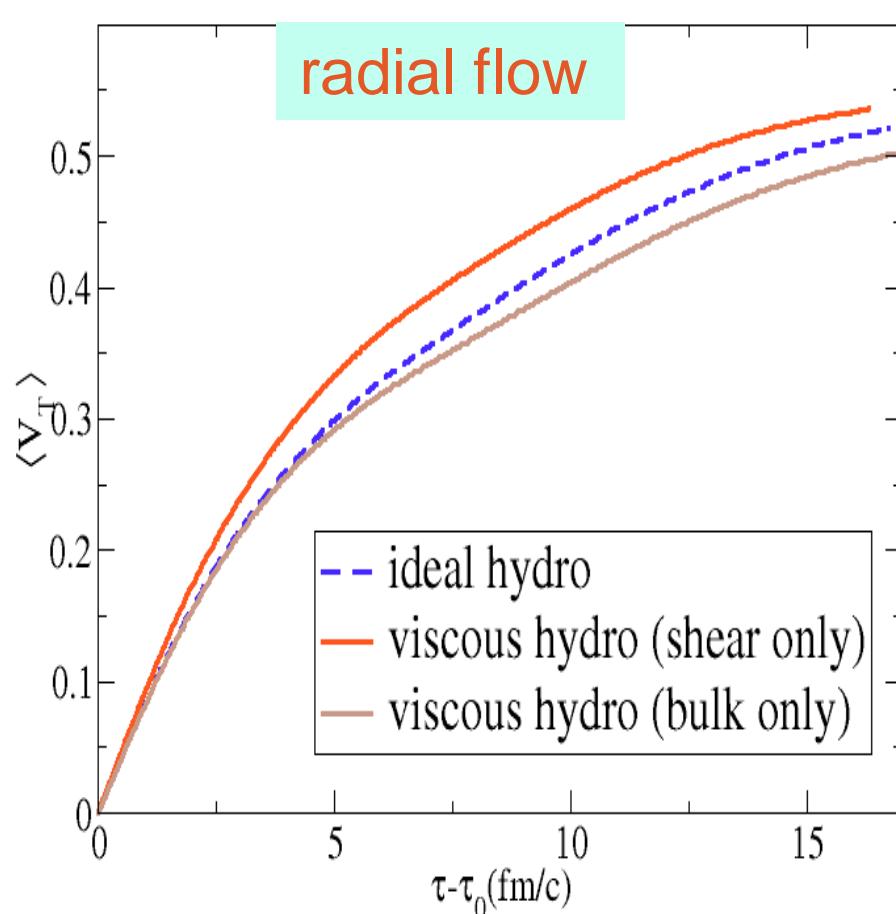
-Shear viscosity: slowing down of cooling process due to decelerated longitudinal expansion initially, but faster cooling in middle and late stages due to stronger transverse expansion

-Bulk viscosity: slowing down of cooling process due to decelerated longitudinal expansion; most of the effects in mixed phase region

# Shear viscosity vs. bulk viscosity (II)

The same Initial & final conditions:

-- dashed ideal hydro    solid viscous hydro-shear only    solid viscous hydro-bulk only



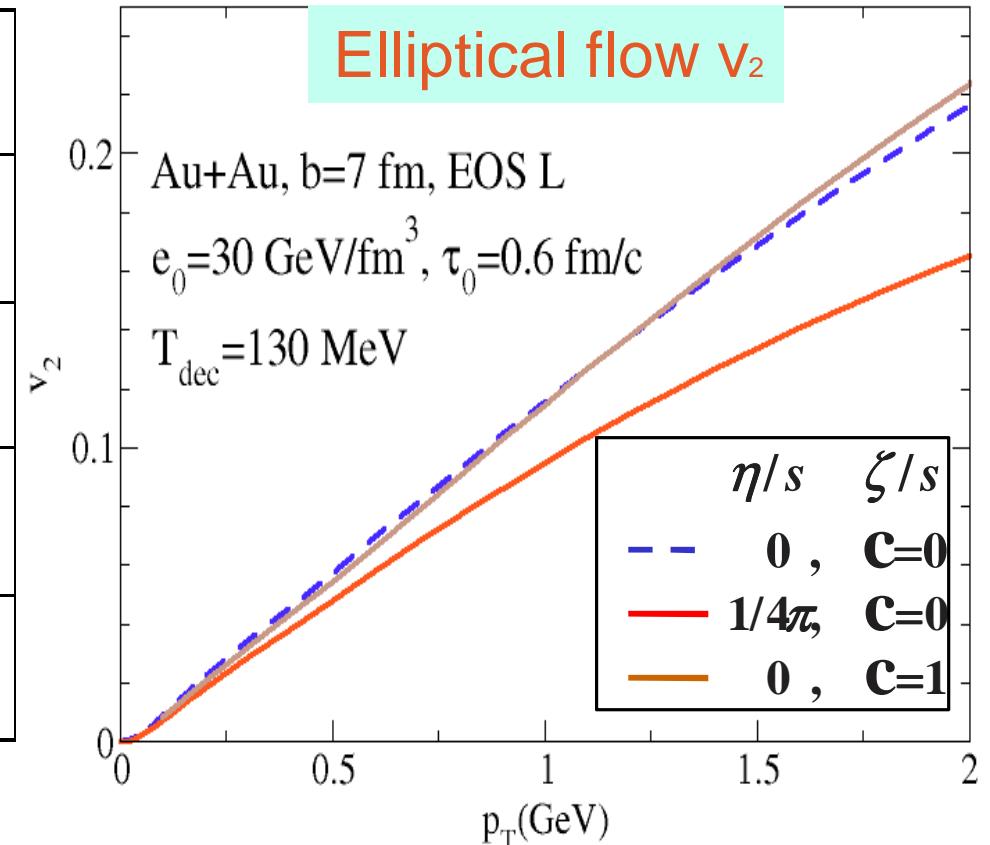
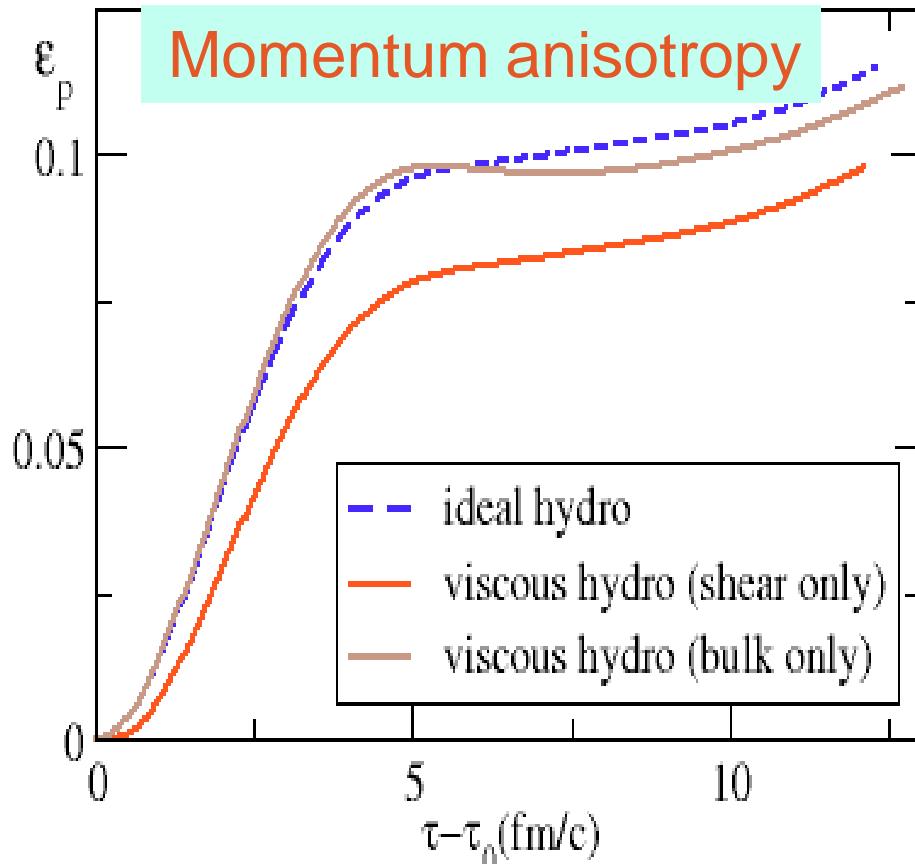
-shear viscosity: increases radial flow, results in flatter spectra

-bulk viscosity: decreases radial flow, results in steeper spectra

# Shear viscosity vs. bulk viscosity (III)

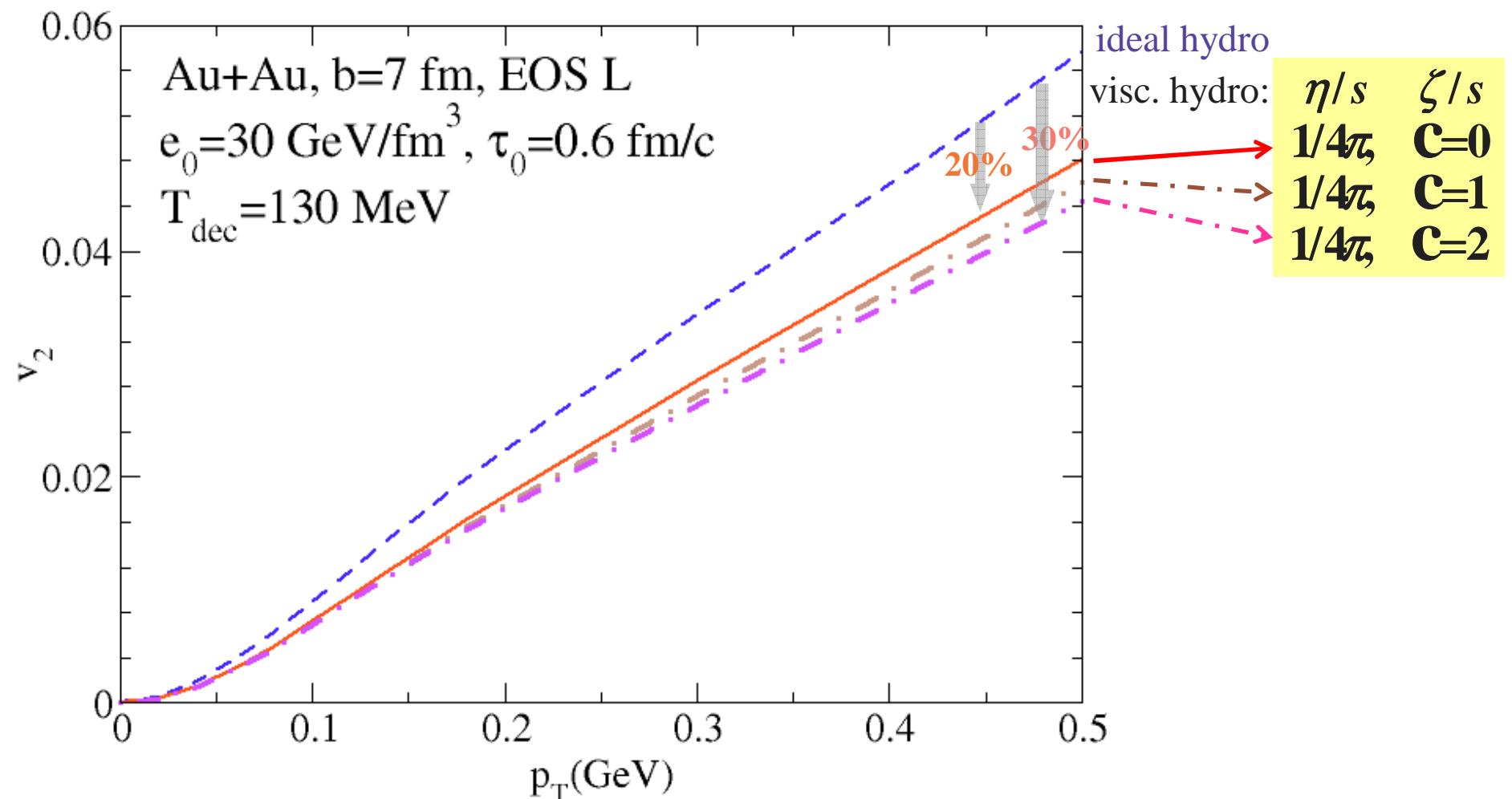
The same Initial & final conditions:

— dashed ideal hydro — viscous hydro-shear only — viscous hydro-bulk only



- both shear and bulk viscosity suppress momentum anisotropy and  $v_2$
- shear viscosity effects concentrate on early stage (QGP phases), bulk viscosity effects only become large during phase transition

# Viscous v<sub>2</sub> suppression: shear and bulk viscosity



- at RHIC, **2 x min. bulk viscosity** could result in **~50%** additional v<sub>2</sub> suppression
- when extracting the  $\eta/s$  from RHIC data, bulk viscous effects cannot be neglected

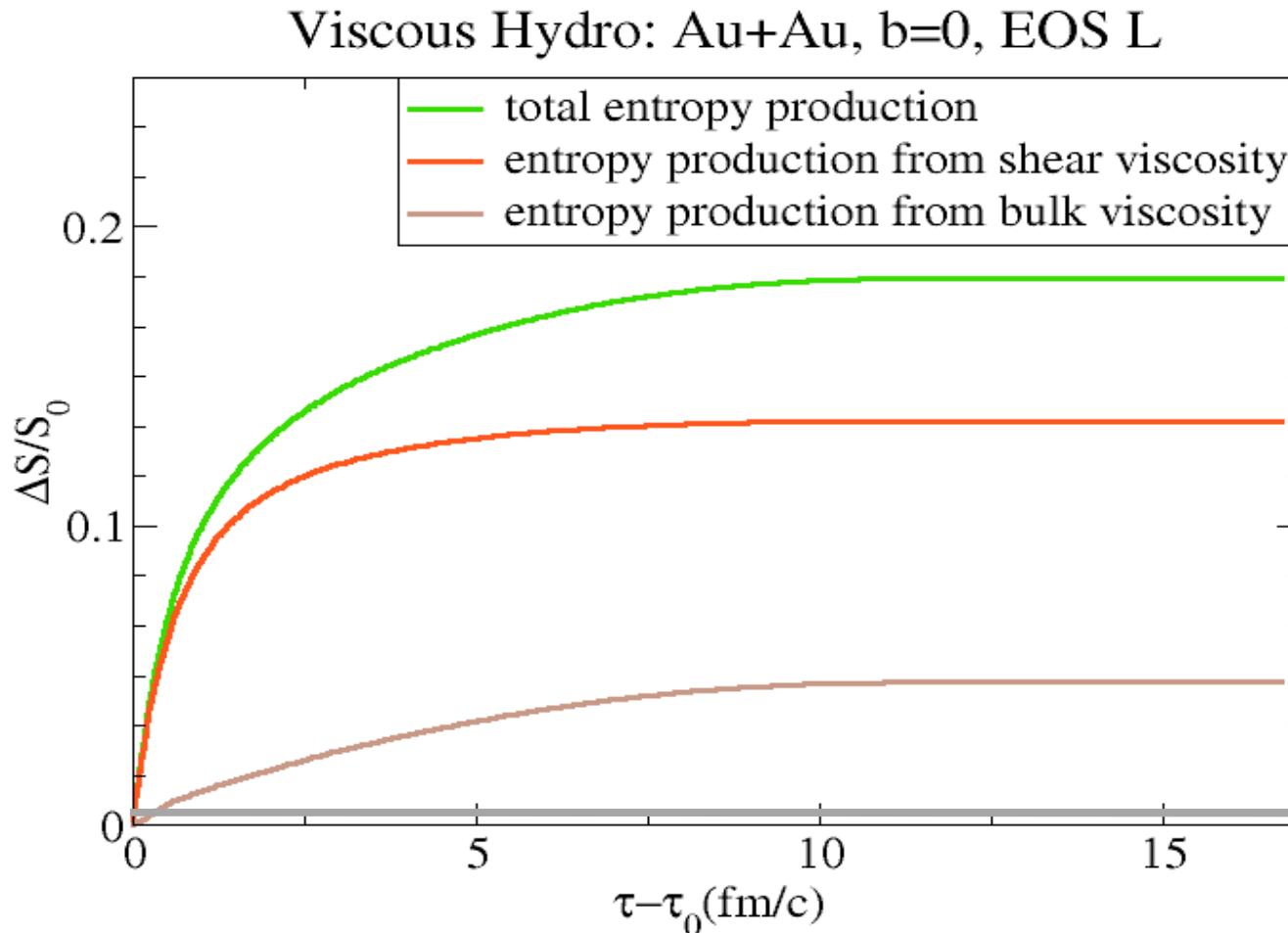
# Numerical Viscosity

-Song & Heinz, PRC 78(2008)  
& figures in TECHQM web pages

To reduce numerical viscosity

- smaller grid sizes
- SHASTLA: anti-diffusion terms

# Entropy Production



typical grid sizes:  
 $dx=dy=0.1\text{fm}$  ,  
 $dt=0.04 \text{ fm/c}$   
anti-diff. const.=0.125

**Real viscosity:**  
 $\Delta S / S_0 \sim 10\%$   
 $\eta/s = 1/4\pi$ ,  $\zeta/s : C = 1$

**Numerical viscosity:**  
 $\Delta S / S_0 \sim 0.3\%$

- Both real viscosity and numerical viscosity generate entropy
- numerical viscosity (with suitable grid sizes and anti-diffusion const.) effects  
are much smaller than real viscosity:

## Numerical Viscosity vs. Real Viscosity

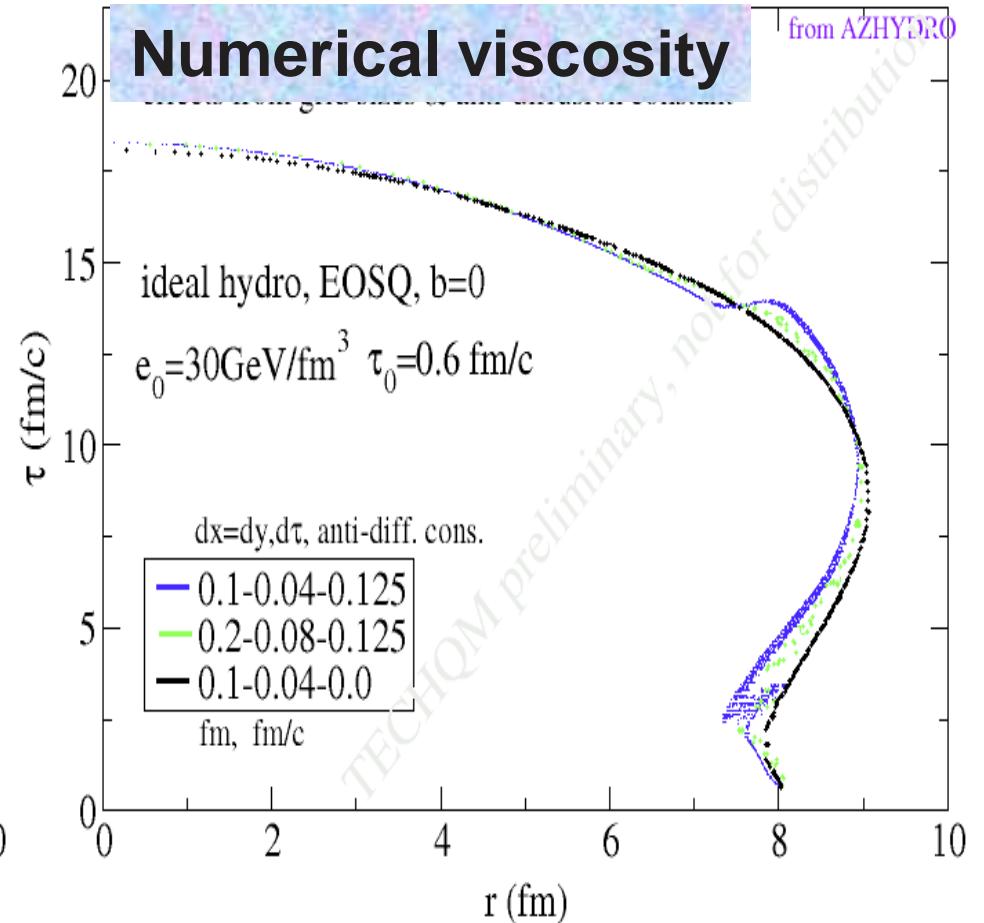
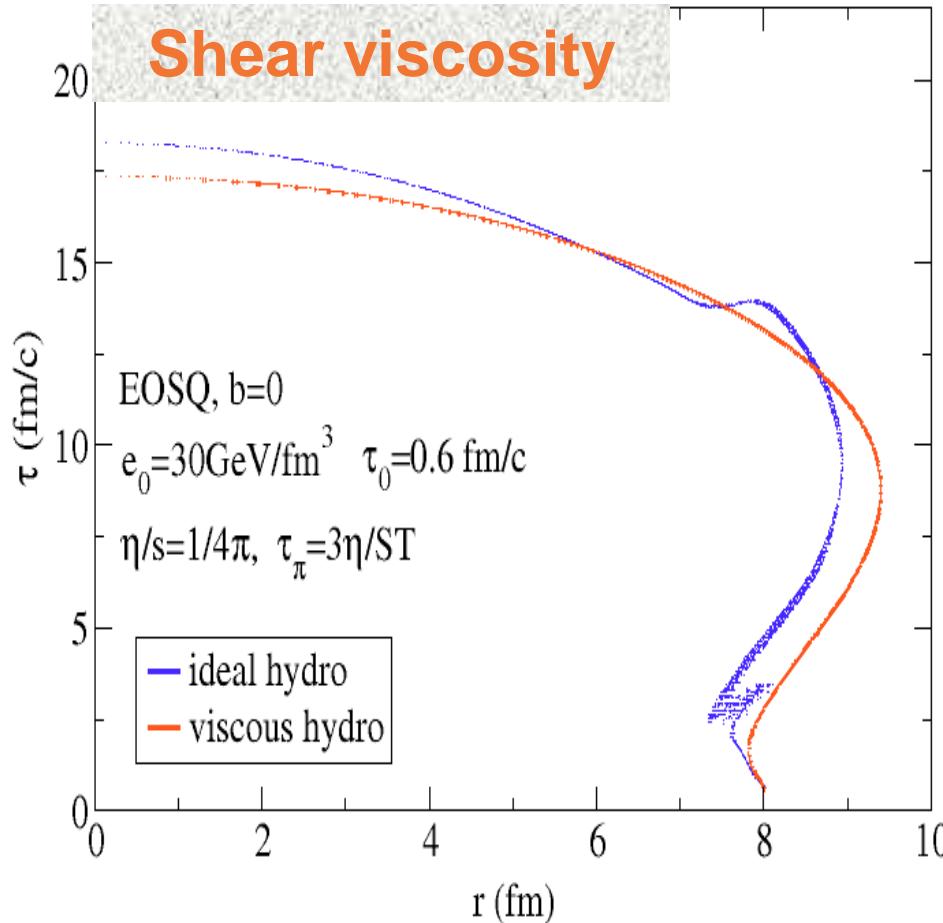
-Song & Heinz, PRC 78(2008)  
& figures in TECHQM web pages

for the effects of larger numerical viscosity:

- larger grid size or zero anti-diff. const.
- EOSQ: 1<sup>st</sup> order phase transition

# Freeze-out surface

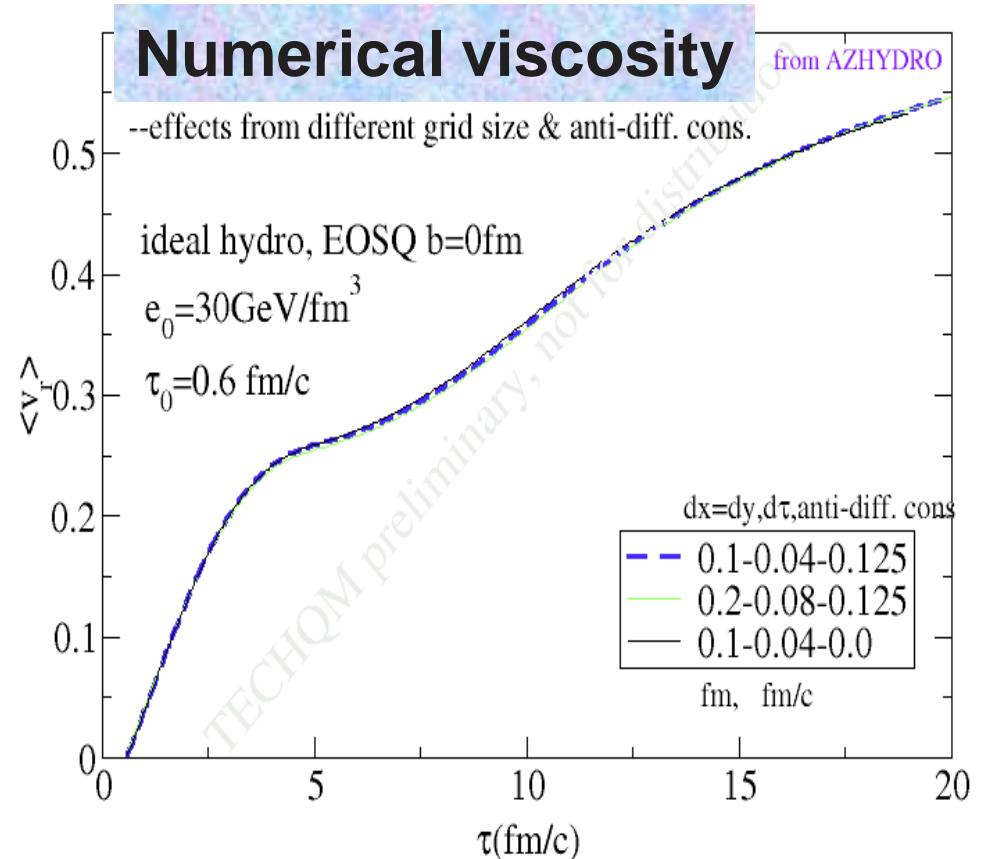
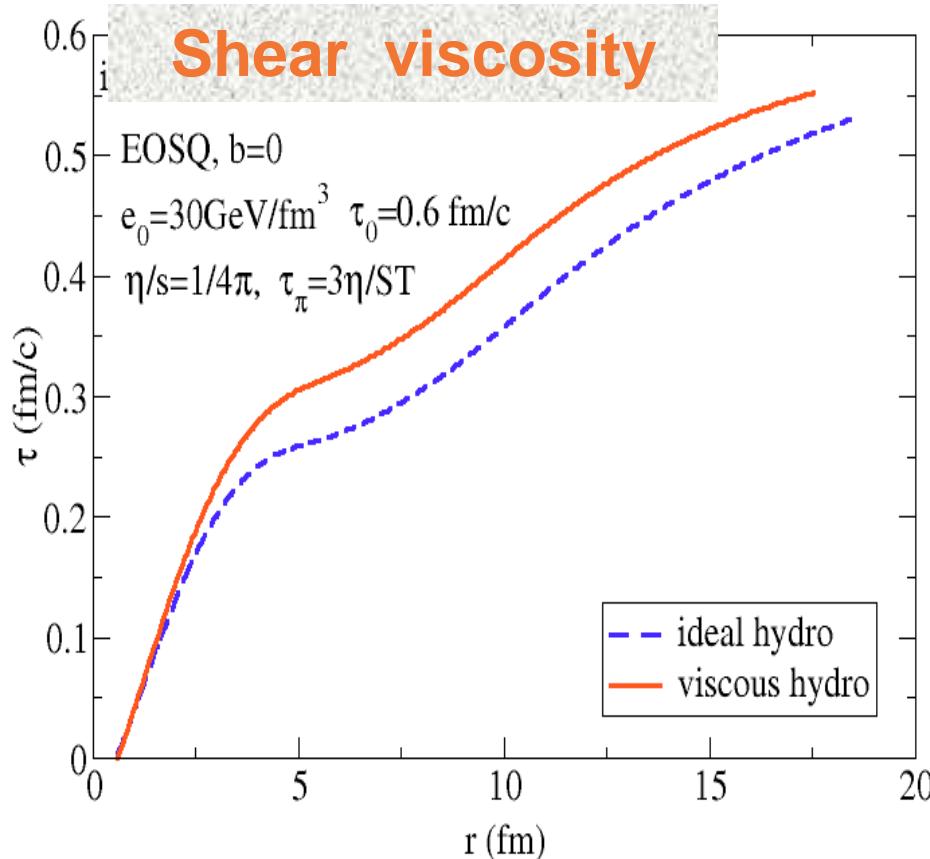
the effects of larger numerical viscosity: larger grid size or zero anti-diff. const.



- both **shear** & **numerical viscosity** smooth-out the phase transition structures
- numerical viscosity less efficient than **shear viscosity** in transverse acceleration

# Radial flow

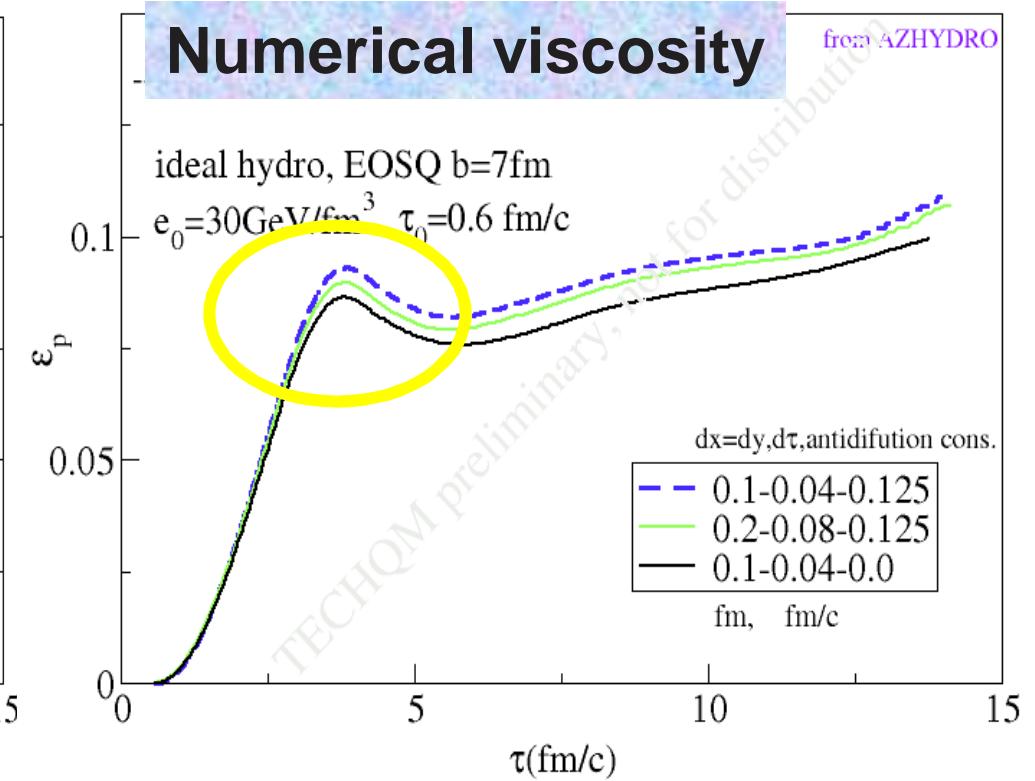
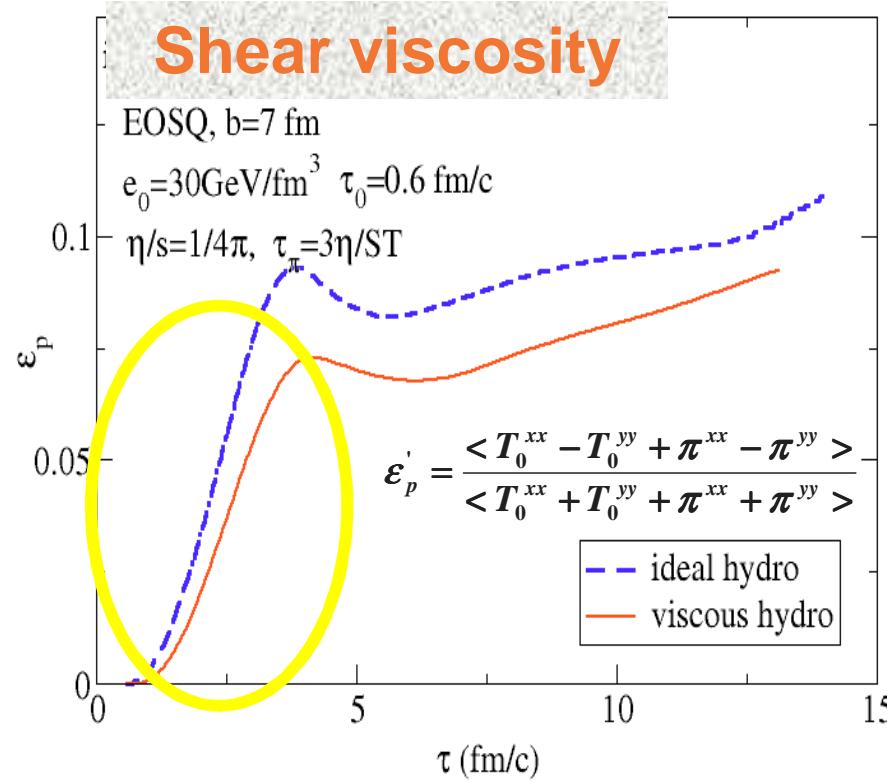
the effects of larger numerical viscosity: larger grid size or zero anti-diff. const.



- shear viscosity accelerates transverse expansion
- numerical viscosity does not obviously accelerate or decelerate transverse expansion

# Momentum anisotropy

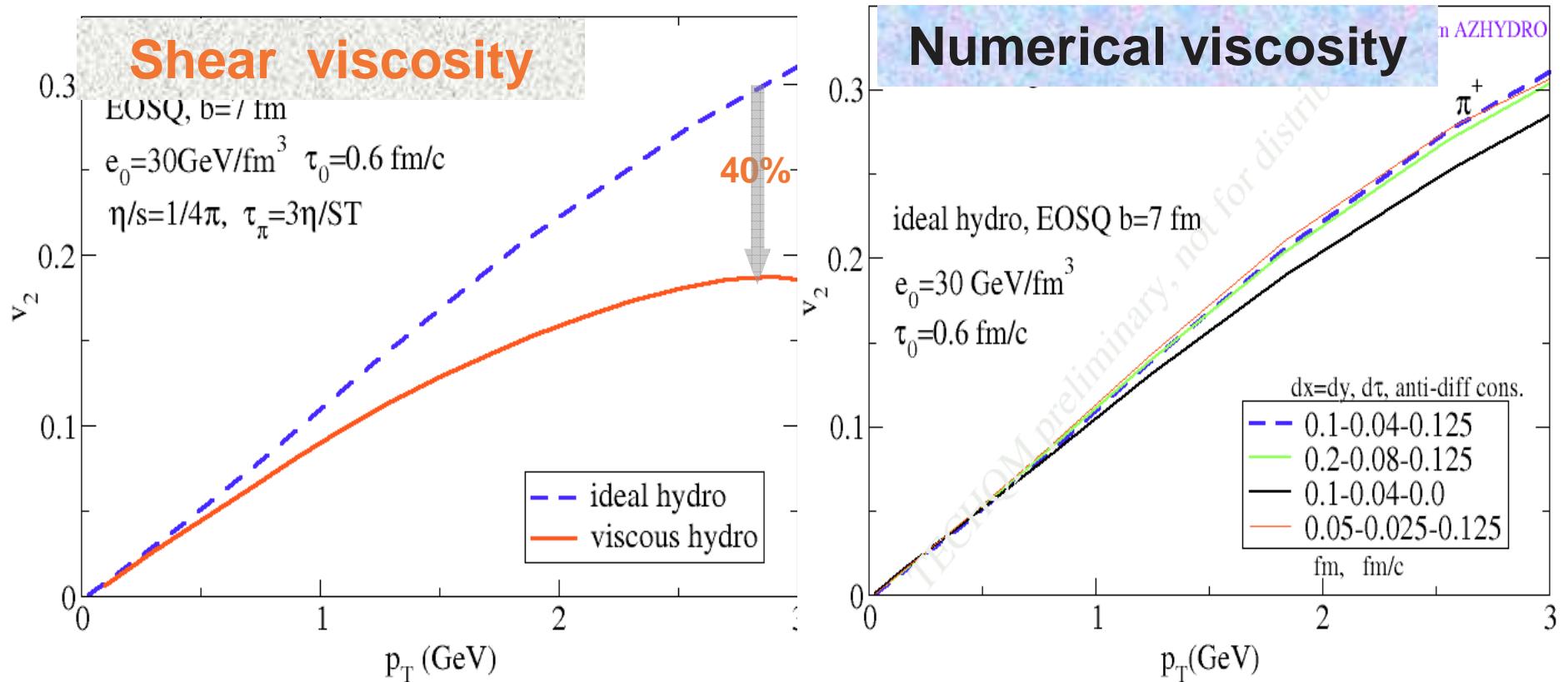
the effects of larger numerical viscosity: larger grid size or zero anti-diff. const.



- Both real viscosity & numerical viscosity inhibit the development of  $\epsilon_p$ 
  - shear viscosity effects concentrate on early time (QGP)
  - numerical viscosity effects concentrate on mixed phase where gradients are large

# Elliptic flow $v_2$

the effects of larger numerical viscosity: larger grid size or zero anti-diff. const.



- Both shear viscosity & numerical viscosity suppress  $v_2$
- for standard settings in VISH2+1 ( $dx=dy=0.1 \text{ fm}$ ,  $dt=0.04 \text{ fm/c}$ ) suppression of  $v_2$  by numerical viscosity is  $< 1\%$

# Summary and discussions

## -Numerical viscosity

- with the typical setting of VISH2+1, the numerical viscosity effects are under control and negligible
- in many aspects real viscosity and numerical viscosity effects are different; one can not fully mimic the effects of real viscosity by using larger numerical viscosity

## -Shear and bulk viscosity

- $\nu_2$  is sensitive to both shear and bulk viscosity
- to extract shear and bulk viscosity separately from experimental data, additional experimental observables must be analyzed

## -to extract QGP viscosity, one must consider (at least) all the following aspects:

- **a realistic EOS:** EOS L vs. SM-EOS Q ~25% (for  $\nu_2$  and  $\nu_2/\varepsilon$ )
- **initial conditions:** CGC vs. Glauber initialization ~15-30% (for  $\nu_2$ )
- **hadronic stage:** viscous hydro+ hadron cascade in the future!

... ...      ... ...

# Thank You

# EOS

